

This is the pre-peer-reviewed version of:

"Market Segmentation Strategies of Multiproduct Firms," Michaela Draganska & Ulrich Doraszelski, 2006.

Journal of Industrial Economics, Blackwell Publishing, vol. 54 (1), pages 125-149.

Published version: <http://www3.interscience.wiley.com/journal/118732644/abstract?CRETRY=1&SRETRY=0>

**Research Paper No. 1827**

**Market Segmentation Strategies of  
Multiproduct Firms**

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**November 2003**

**RESEARCH PAPER SERIES**

**STANFORD**  
GRADUATE SCHOOL OF BUSINESS



# Market Segmentation Strategies of Multiproduct Firms \*

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November 26, 2003

## Abstract

We analyze a multiproduct duopoly and ask whether firms should offer general purpose products or tailor their offerings to fit specific consumer needs. There are two effects of offering a targeted product: (i) if a consumer's favorite product is offered, her utility increases because there is a better fit between product and preferences; (ii) if her favorite product is not offered, the consumer's utility decreases because she gets a product that is not tailored to her needs at all. Previous work has not considered these two effects jointly and has therefore not been able to capture the tradeoff inherent in market segmentation: for some consumers utility increases due to increased "fit" whereas for others utility decreases due to increased "misfit." We show that in addition to the degree of fit and misfit, the intensity of competition and the fixed cost of offering an additional product determine firms' market segmentation strategies.

*Key Words:* multiproduct firms, market segmentation, competitive strategy.

*JEL Classification:* L1; M3

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\*Thanks to Scott Davis, Dino Gerardi, Wes Hartmann, Ken Judd, Ted Turocy, and Katja Seim for helpful comments and suggestions. Liang Qiao provided excellent research assistance.

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# 1 Introduction

Extending product lines to target more narrowly defined consumer segments has been a favorite strategy of brand managers for years. Popular buzzwords like “one-on-one marketing” and “niche marketing” underline its importance in practice. In this context, we ask to what extent market segmentation is justified from a profitability point of view.

Consider a competitive environment in which each firm can either offer a general purpose product or segment the market by offering products that are tailored to consumers’ needs. While a consumer *ceteris paribus* prefers a product that is targeted at her own segment, she would rather buy a general purpose product than a product that is targeted at another segment. Segmenting the market therefore has two effects. First, if a consumer’s preferred product is offered, she is better off because she is able to buy a product that exactly fits her needs. We call this positive aspect of market segmentation “fit.” Second, if the consumer’s preferred product is not offered, she is worse off because she ends up with a product that does not satisfy her needs at all. This negative aspect of market segmentation is denoted as “misfit.”

For example, in the sport shoe industry a firm can either offer a general purpose cross-trainer shoe or a running shoe and a basketball shoe. A cross-trainer shoe has wide appeal for all consumers but satisfies no consumer’s needs in particular while the specific shoes each satisfy the needs of a particular segment of consumers but have little appeal for the other segment. On the other hand, the firm can offer both a running shoe and a basketball shoe. Manufacturing a larger number of products, however, carries a greater fixed cost than manufacturing a smaller number of products. Whether the firm will follow a niche or a full-line strategy therefore depends on the cost of offering an additional product and the revenue generated by doing so. The

revenue in turn depends on the intensity of competition in the market.

In this paper we therefore investigate how consumers' preferences, firms' cost structures, and the strategic interaction of firms in the product market together shape firms' market segmentation decisions. There are two streams of literature that are related to our research: the literature on spatial competition and the one on multi-product firms. We discuss each of them in turn.

In models of spatial competition misfit or "specificity" is operationalized by transportation costs and fit or "quality" by gross benefits. The existing research has studied these two aspects of market segmentation separately from each other. For example, von Ungern-Sternberg (1988) considers  $n$  single-product firms and analyzes a two-stage game in which entry decisions are followed by choice of transportation costs and prices. He finds that the private incentives to produce general purpose products (i.e., lower transportation costs) are excessive relative to the social optimum. Hendel & Neiva de Figueiredo (1998) argue that von Ungern-Sternberg's (1988) timing prevents the choice of transportation costs to have a strategic effect on firms' pricing and hence propose a three-stage game: entry decisions, followed by choice of transportation costs, and then choice of prices. Their results suggest that when the degree of specificity can be varied without affecting firms' costs, then at most two firms will enter the market and choose positive transportation costs. The results for the case where general purposeness is costly are inconclusive.

Rather than focusing on endogenous transportation costs, Economides (1989, 1993) analyzes endogenous gross benefits and interprets them as quality. Economides (1993) finds that there is too much variety and too little quality in a free-entry equilibrium. Unlike the above models, which all consider competition on a circle, Economides (1989) has consumers located along a line. He explicitly considers firms' location decisions and shows that the ensuing equilibrium has maximum location

differentiation but minimum quality differentiation.

None of these models captures the tradeoff between the increased fit for some consumers and the increased misfit for others that is inherent in market segmentation. More important, these models are ill-suited to study firms' market segmentation decisions because each firm is constrained to offer a single product.

Brander & Eaton (1984) first analyzed competition between multiproduct duopolists. Constraining each firm to sell exactly two products and fixing the attributes of the four products in the market, they ask whether a firm would decide to produce a pair of close substitutes ("market segmentation") or a pair of distant substitutes ("market interlacing"). Their assumption that a firm has to offer two products has subsequently been criticized. In particular, Martinez-Giralt & Neven (1988) argue that when product attributes are endogenized, price competition gives rise to Hotelling's principle of maximum differentiation and *de facto* prevents multiproduct firms. In the context of the sport shoe industry this suggests an outcome of market segmentation with niche firms, i.e., one firm offers a running shoe, the other a basketball shoe. What we observe, however, is that in reality a number of firms compete head-on by offering multiple products.

To investigate firms' market segmentation strategies in a competitive setting, we propose a simple model of a multiproduct duopoly. There are two segments of consumers, and each firm has a choice between offering a general purpose product and tailoring its offerings to one or both consumer segments. In contrast to Brander & Eaton's (1984) model, we thus let a firm choose both the number and type of products to offer. To keep the model tractable we assume, similar to Brander & Eaton (1984), that product attributes are given. We, however, add two parameters that enable us to capture the tradeoff between general purpose and tailored products in this setting. One parameter represents fit, i.e., how much a consumer may gain from

market segmentation, the other misfit, i.e., how much the consumer may lose. This parameterization enables us to extend the ideas behind models of spatial competition with endogenous transportation costs/gross benefits to multiproduct firms.

In contrast to models of spatial competition, we allow consumers to have idiosyncratic preferences for firms. The stronger these brand preferences are, the softer the competition between firms is. Hence, even head-on competition generates positive profits, and a firm may find it in its best interest to duplicate the product offerings of its rival. Incorporating idiosyncratic brand preferences allows us to escape the unrealistic implication of models of spatial competition that firms will never compete head-on, say by both offering a general purpose product.

We identify four key determinants of market segmentation, namely the degree of fit, the degree of misfit, the intensity of competition, and the fixed cost of offering an additional product. We provide conditions under which (i) in equilibrium both firms forego the possibility of segmenting the market by offering a general purpose product; (ii) market segmentation occurs via niche firms; and (iii) market segmentation occurs via full-line firms. Our model is thus able to account for a variety of outcomes that we observe in the marketplace.

The remainder of this paper is organized as follows: Section 2 sets up the model. We characterize the equilibrium in Section 3 and summarize our results in Section 4. Section 5 concludes.

## 2 Model

There are two firms, 1 and 2, and two segments of heterogeneous consumers,  $a$  and  $b$ . Each firm chooses between the following offerings:

- the general purpose product  $GP$ ;

- product  $A$  which is targeted at segment  $a$ ;
- product  $B$  which is targeted at segment  $b$ ;
- or products  $A$  and  $B$  (denoted as  $AB$ ).

That is, a firm that chooses to segment the market can be a niche firm and offer either  $A$  or  $B$ , or it can be a full-line firm and offer both  $A$  and  $B$ . Firms compete in prices. We assume that if a firm offers more than one product, it charges the same price for all its products. This assumption simplifies the analysis considerably and is also satisfied or at least a good approximation to observed behavior in many product categories.

The marginal cost of production is  $c \geq 0$ . The fixed cost of production is  $f \geq 0$  per product. That is, we assume that manufacturing a larger number of products carries a greater fixed cost than manufacturing a smaller number of products. Moreover, we take the cost of the general purpose product to be the same as the cost of the tailored products. In contrast, von Ungern-Sternberg (1988) and Hendel & Neiva de Figueiredo (1998) assume that producing a general purpose product is more costly than producing a specific product, an *ad-hoc* assumption that is needed in their case to ensure an interior solution to a firm's profit maximization problem.

Consumers have unit demand. Consider a consumer in segment  $a$  who is looking at the products of firm 1. Her utility from the general purpose product at price  $p_1$  is

$$v - p_1 + \epsilon_1,$$

where  $v$  is the gross benefit that the consumer derives from the general purpose product and  $\epsilon_1$  is the consumer's idiosyncratic preference for firm 1. This idiosyncratic preference may represent her taste for a brand name or company reputation. The

consumer's utility from product  $A$  is

$$\bar{v} - p_1 + \epsilon_1$$

and her utility from product  $B$  is

$$\underline{v} - p_1 + \epsilon_1.$$

Similarly, the utility of a consumer in segment  $b$  who is looking at the products of firm 1 is  $v - p_1 + \epsilon_1$  for the general purpose product,  $\underline{v} - p_1 + \epsilon_1$  for product  $A$ , and  $\bar{v} - p_1 + \epsilon_1$  for product  $B$ . The utility that a consumer derives from the products of firm 2 is given by analogous expressions. We assume

$$\underline{v} < v < \bar{v}.$$

That is, a consumer in segment  $a$  ( $b$ ) receives the highest gross benefit from product  $A$  ( $B$ ), which is targeted at her own segment, and the lowest gross benefit from product  $B$  ( $A$ ), which is targeted at the other segment. The gross benefit of the general purpose product is somewhere between the ones of products  $A$  and  $B$ . Hence, a consumer gains  $\bar{v} - v$  from market segmentation if her preferred product is offered and loses  $\underline{v} - v$  if her preferred product is not offered. In other words,  $\bar{v} - v$  measures the *utility gain* from market segmentation due to the *increased fit* and  $\underline{v} - v$  the *utility loss* due to the *increased misfit*.

We assume that the market is fully covered and that each segment has a unit mass of consumers. We further assume that  $\epsilon_1 \sim F$ , where  $F(\epsilon_1) = 1 - F(-\epsilon_1)$ , and that  $\epsilon_2 = 0$ , where  $\epsilon_2$  is the idiosyncratic preference for firm 2. This implies that if  $\epsilon_1 > 0$  ( $\epsilon_1 < 0$ ), then the consumer is biased in favor of firm 1 (firm 2), and that these

idiosyncratic preferences are symmetric across firms.

Our parameters  $\underline{v}$ ,  $v$ , and  $\bar{v}$  can be viewed as combining transportation costs and gross benefits in traditional models of spatial competition. To see this, suppose that consumers in segments  $a$  and  $b$  are located at opposite ends of a Hotelling line of unit length and that the tailored products are located at the corresponding ends, whereas the general purpose product is located in the middle. Suppose further that gross benefits are different for the tailored products and the general purpose product. Then the utility (before price) that a consumer in segment  $a$  obtains from buying product  $A$  is  $\nu^{A,B}$ , where  $\nu^{A,B}$  is the gross benefit of the tailored products. Note that in this case the transportation costs are zero. The utility that this consumer obtains from buying product  $B$  ( $GP$ ) is  $\nu^{A,B} - \tau (\nu^{GP} - \frac{\tau}{2})$ , where  $\nu^{GP}$  is the gross benefit of the general purpose product and  $\tau$  is the transportation cost per unit of distance. Hence,  $\underline{v} = \nu^{A,B} - \tau$ ,  $v = \nu^{GP} - \frac{\tau}{2}$ , and  $\bar{v} = \nu^{A,B}$ . In this sense, by allowing a firm to choose the number and type of products to offer, we endogenize both transportation costs and gross benefits.

We consider a two-stage game. The timing is as follows:

1. Firms simultaneously decide on their product offerings.
2. Price competition takes place.

We look for the subgame perfect equilibria (SPEs) of this two-stage game.

### 3 Equilibrium

We solve for SPEs by backwards induction.

**Price competition.** Suppose that firm 1 offers  $GP$  and firm 2 offers  $GP$ . Then a consumer in segment  $a$  buys from firm 1 if

$$v - p_1 + \epsilon_1 \geq v - p_2 \Leftrightarrow \epsilon_1 \geq p_1 - p_2$$

and a consumer in segment  $b$  buys from firm 1 if

$$v - p_1 + \epsilon_1 \geq v - p_2 \Leftrightarrow \epsilon_1 \geq p_1 - p_2.$$

Hence, total demand for firm 1 is  $2(1 - F(p_1 - p_2)) = 2F(p_2 - p_1)$  and total demand for firm 2 is  $2F(p_1 - p_2)$ . Since firms offer the same product, consumers in both segments base their decisions solely on the difference in prices and their idiosyncratic preferences for firms.

Suppose next that firm 1 offers  $GP$  and firm 2 offers  $A$ . Then a consumer in segment  $a$  buys from firm 1 if

$$v - p_1 + \epsilon_1 \geq \bar{v} - p_2 \Leftrightarrow \epsilon_1 \geq \bar{v} - v + p_1 - p_2$$

and a consumer in segment  $b$  buys from firm 1 if

$$v - p_1 + \epsilon_1 \geq \underline{v} - p_2 \Leftrightarrow \epsilon_1 \geq \underline{v} - v + p_1 - p_2.$$

Hence, total demand for firm 1 is  $1 - F(\bar{v} - v + p_1 - p_2) + 1 - F(\underline{v} - v + p_1 - p_2) = F(v - \bar{v} + p_2 - p_1) + F(v - \underline{v} + p_2 - p_1)$  and total demand for firm 2 is  $F(\bar{v} - v + p_1 - p_2) + F(\underline{v} - v + p_1 - p_2)$ . Since firms no longer offer the same product, consumers take not only the difference in prices but also the difference in gross benefits into account when making their purchase decisions.

Finally suppose that firm 1 offers  $GP$  and firm 2 offers  $A$  and  $B$ . Then a consumer in segment  $a$  buys from firm 1 if

$$v - p_1 + \epsilon_1 \geq \max\{\bar{v} - p_2, \underline{v} - p_2\} \Leftrightarrow \epsilon_1 \geq \bar{v} - v + p_1 - p_2$$

and a consumer in segment  $b$  buys from firm 1 if

$$v - p_1 + \epsilon_1 \geq \max\{\underline{v} - p_2, \bar{v} - p_2\} \Leftrightarrow \epsilon_1 \geq \bar{v} - v + p_1 - p_2.$$

Hence, total demand for firm 1 is  $2(1 - F(\bar{v} - v + p_1 - p_2)) = 2F(v - \bar{v} + p_2 - p_1)$  and total demand for firm 2 is  $2F(\bar{v} - v + p_1 - p_2)$ .

In general, total demand for firm 1 is of the form  $F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1)$  and total demand for firm 2 is of the form  $F(-\Delta_a + p_1 - p_2) + F(-\Delta_b + p_1 - p_2)$ .  $\Delta_a > 0$  implies that firm 1's product offerings better satisfy the needs of segment  $a$  than firm 2's product offerings. Similarly,  $\Delta_b$  captures the match between firm 1's products and segment  $b$ 's needs relative to firm 2's products. Table 1 lists  $(\Delta_a, \Delta_b)$  for all possible combinations of product offerings.<sup>1</sup>

1\2	$GP$	$A$	$B$	$AB$
$GP$	$(0, 0)$	$(v - \bar{v}, v - \underline{v})$	$(v - \underline{v}, v - \bar{v})$	$(v - \bar{v}, v - \bar{v})$
$A$	$(\bar{v} - v, \underline{v} - v)$	$(0, 0)$	$(\bar{v} - \underline{v}, \underline{v} - \bar{v})$	$(0, \underline{v} - \bar{v})$
$B$	$(\underline{v} - v, \bar{v} - v)$	$(\underline{v} - \bar{v}, \bar{v} - \underline{v})$	$(0, 0)$	$(\underline{v} - \bar{v}, 0)$
$AB$	$(\bar{v} - v, \bar{v} - v)$	$(0, \bar{v} - \underline{v})$	$(\bar{v} - \underline{v}, 0)$	$(0, 0)$

Table 1:  $(\Delta_a, \Delta_b)$  for all possible combinations of product offerings. Firm 1 is row, firm 2 is column.

Given product offerings  $(\Delta_a, \Delta_b)$ , gross profits (before fixed costs are subtracted)

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<sup>1</sup>Our assumption that a firm charges the same price for all its products has bite only if one firm offers both  $A$  and  $B$  and the other either  $A$  or  $B$  and ensures that we can continue to write total demand of, say, firm 1, as  $F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1)$ . Because of symmetry this assumption does not constrain firms' pricing behavior in any other situation.

are

$$\begin{aligned}\pi_1(p_1, p_2) &= \left( F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1) \right) (p_1 - c), \\ \pi_2(p_1, p_2) &= \left( F(-\Delta_a + p_1 - p_2) + F(-\Delta_b + p_1 - p_2) \right) (p_2 - c),\end{aligned}$$

and the first two derivatives of  $\pi_1(p_1, p_2)$  with respect to  $p_1$  are

$$\begin{aligned}\frac{\partial}{\partial p_1} \pi_1(p_1, p_2) &= - \left( F'(\Delta_a + p_2 - p_1) + F'(\Delta_b + p_2 - p_1) \right) (p_1 - c) \\ &\quad + F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1), \\ \frac{\partial^2}{\partial p_1^2} \pi_1(p_1, p_2) &= \left( F''(\Delta_a + p_2 - p_1) + F''(\Delta_b + p_2 - p_1) \right) (p_1 - c) \\ &\quad - 2 \left( F'(\Delta_a + p_2 - p_1) + F'(\Delta_b + p_2 - p_1) \right).\end{aligned}$$

Our next task is to characterize the Nash equilibrium (NE) of the pricing subgame. To this end, we assume that  $\epsilon_1$  is logistically distributed with  $F(\epsilon_1) = \frac{1}{1 + e^{-\frac{\epsilon_1}{\beta}}}$ , where  $\beta > 0$  is proportional to the standard deviation of the idiosyncratic shock and measures how sensitive market shares are to differences in product offerings and prices between the two firms. Indeed, as  $\beta$  grows, consumers become more prone to buy from their favorite firm irrespective of product offerings and prices. We thus say that consumers have strong (weak) *brand preferences* if  $\beta$  is high (low). Consequently,  $\beta$  is also a measure of the *intensity of competition*: if  $\beta$  is high, competition is soft, and if  $\beta$  is low, competition is fierce.

Using the properties of the logistic distribution, we have  $F'(\epsilon_1) = \frac{1}{\beta} F(\epsilon_1)(1 - F(\epsilon_1))$  and  $F''(\epsilon_1) = \frac{1}{\beta^2} F(\epsilon_1)(1 - F(\epsilon_1))(1 - 2F(\epsilon_1))$ . In the special case of  $\Delta_a = \Delta_b$ , the product offerings of, say, firm 1 are equally suited to both segments of consumers. From the firm's perspective it is thus as if there were just one large segment. It is now straightforward to show that  $\pi_1(p_1, p_2)$  is strictly quasiconcave in  $p_1$ , so that results

by Caplin & Nalebuff (1991) imply that there exists a unique NE of the pricing subgame (see also Chapters 6 and 7 of Anderson, de Palma & Thisse (1992) and Mizuno (2003)). While no analytic solution is available, the NE can be computed by numerically solving the system of FOCs given by  $\frac{\partial \pi_1}{\partial p_1}(p_1, p_2) = 0$  and  $\frac{\partial \pi_2}{\partial p_2}(p_1, p_2) = 0$ .

Unfortunately, these results no longer apply if  $\Delta_a \neq \Delta_b$ , and there may in fact not be a NE in pure strategies. To see this, suppose that firm 1's product offerings satisfy the needs of segment  $a$  well,  $\Delta_a \gg 0$ , and the needs of segment  $b$  poorly,  $\Delta_b \ll 0$ . Then the firm faces a choice between exploiting the consumers in segment  $a$  by setting a high price and setting a low price in order to be competitive in segment  $b$ . This may give rise to a discontinuity in the firm's best reply function and ultimately lead to nonexistence of a NE in pure strategies. By verifying that none of the two firms has a profitable unilateral deviation, however, it is easy to ensure that a numerical solution to the system of FOCs constitutes a NE of the pricing subgame.

Figure 1 depicts the equilibrium market shares of firm 1 in segments  $a$  and  $b$ , its price, and its profit as a function of  $(\Delta_a, \Delta_b)$ . The equilibrium price of firm 2 is given by  $p_2^*(\Delta_a, \Delta_b) = p_1^*(-\Delta_a, -\Delta_b)$ , its equilibrium profit by  $\pi_2^*(\Delta_a, \Delta_b) = \pi_1^*(-\Delta_a, -\Delta_b)$ . Figure 1 is based on  $\beta = 0.5$  and  $c = 0$ . These parameter values imply that competition is rather fierce, with the elasticity of firm 1's demand with respect to firm 2's price ranging from 0.32 at  $(-2, -2)$  to 3.16 at  $(2, 2)$ . As can be seen, for some  $(\Delta_a, \Delta_b) \in [-2, 2]^2$ , there is no NE of the pricing subgame. One intuitively suspects that existence becomes less problematic as competition becomes less intense, and our computations readily confirm this. For example, if we increase  $\beta$  from 0.5 to 1, then there is a NE for all product offerings.

Contrary to what one may expect,  $p_1^*(\Delta_a, \Delta_b)$  and  $\pi_1^*(\Delta_a, \Delta_b)$  are not necessarily increasing in  $\Delta_a$  and  $\Delta_b$ . That is, a better match between a firm's products and consumer's needs does not directly translate into higher prices and profits. Consider

again a situation in which firm 1's product offerings satisfy the needs of segment  $a$  well,  $\Delta_a \gg 0$ , and the needs of segment  $b$  poorly,  $\Delta_b \ll 0$ . Then an increase in  $\Delta_b$  leads *ceteris paribus* to a large increase in firm 1's share of segment  $b$ . Firm 2 reacts to this large decrease in its share of segment  $b$  by dropping its price sharply which in turn forces firm 1 to drop its price. Overall, the total effect on firm 1's profit is negative. On the other hand, consider a situation in which firm 1's product offerings satisfy the needs of both segments well,  $\Delta_a \gg 0$  and  $\Delta_b \gg 0$ . Then an increase in  $\Delta_b$  leads *ceteris paribus* to a small increase in firm 1's share of segment  $b$ . Firm 2 reacts to this small decrease in its share of segment  $b$  by dropping its price marginally. This leaves firm 1 room for a price increase. Overall, the total effect on firm 1's profit is positive.

More generally, as the following proposition shows,  $p_1^*(\Delta_a, \Delta_b)$  and  $\pi_1^*(\Delta_a, \Delta_b)$  are increasing in  $\Delta_a$  and  $\Delta_b$  provided that  $\Delta_a \approx \Delta_b$ , i.e., provided that the product offerings of both firms are similarly suited to both segments of consumers. It follows immediately that  $p_2^*(\Delta_a, \Delta_b)$  and  $\pi_2^*(\Delta_a, \Delta_b)$  are decreasing in  $\Delta_a$  and  $\Delta_b$  provided that  $\Delta_a \approx \Delta_b$ .

**Proposition 1** *Suppose  $\Delta_a \approx \Delta_b$ . Then  $\frac{\partial p_1^*}{\partial \Delta_a} > 0$ ,  $\frac{\partial p_1^*}{\partial \Delta_b} > 0$ ,  $\frac{\partial \pi_1^*}{\partial \Delta_a} > 0$ , and  $\frac{\partial \pi_1^*}{\partial \Delta_b} > 0$ .*

**Proof.** Totally differentiate the system of FOCs. Use the fact that  $\Delta_a = \Delta_b$  implies  $F(\Delta_a + p_2^* - p_1^*) = F(\Delta_b + p_2^* - p_1^*)$  along with  $F(\epsilon_1) = 1 - F(-\epsilon_1)$  to obtain

$$\begin{aligned} \frac{\partial p_1^*}{\partial \Delta_a} = \frac{\partial p_1^*}{\partial \Delta_b} &= \frac{F(\Delta_a + p_2^* - p_1^*)^2}{2(1 - F(\Delta_a + p_2^* - p_1^*) + F(\Delta_a + p_2^* - p_1^*)^2)} > 0, \\ \frac{\partial p_2^*}{\partial \Delta_a} = \frac{\partial p_2^*}{\partial \Delta_b} &= \frac{-(1 - F(\Delta_a + p_2^* - p_1^*))^2}{2(1 - F(\Delta_a + p_2^* - p_1^*) + F(\Delta_a + p_2^* - p_1^*)^2)} < 0, \end{aligned}$$

where we also use the fact that the FOCs can be rewritten as  $\frac{p_1^* - c}{\beta} = \frac{1}{1 - F(\Delta_a + p_2^* - p_1^*)}$

and  $\frac{p_2^* - c}{\beta} = \frac{1}{F(\Delta_a + p_2^* - p_1^*)}$  to substitute for the price-cost margins. It follows that

$$\frac{\partial \pi_1^*}{\partial \Delta_a} = \frac{\partial \pi_1^*}{\partial \Delta_b} = \frac{F(\Delta_a + p_2^* - p_1^*)^2}{1 - F(\Delta_a + p_2^* - p_1^*) + F(\Delta_a + p_2^* - p_1^*)^2} > 0.$$

The proof is completed by noting that continuity implies that the above carries over to a neighborhood of  $\Delta_a = \Delta_b$ . ■

**Product offerings.** The stage game is finite. Provided that there exists a NE in all relevant pricing subgames, the overall game has at least one SPE (possibly in mixed strategies). Our goal is to characterize the set of SPEs and to describe how it changes as the parameters  $\underline{v}$ ,  $v$ ,  $\bar{v}$ ,  $\beta$ ,  $c$ , and  $f$  change.

The simplicity of our model allows us to reduce the number of parameters, thereby aiding the subsequent analysis. First observe from Table 1 that only differences between  $\bar{v}$ ,  $v$ , and  $\underline{v}$  matter for determining total demand and hence firms' profits. We can therefore set  $v = 0$  without loss of generality. This implies that  $\bar{v} > 0$  ( $\underline{v} < 0$ ) measures the gross benefit a consumer gains (loses) from buying a product that is targeted at her own segment (the other segment) instead of the general purpose product. In other words,  $\bar{v}$  ( $\underline{v}$ ) measures the utility gain (loss) due to the *increased fit (misfit)*, and we henceforth interpret  $\bar{v}$  ( $|\underline{v}|$ ) as a measure of the degree of fit (misfit). Note next that the marginal cost of production determines the extent of the transfer from consumers to firms but has no impact on firms' profits in equilibrium. We thus set  $c = 0$ . Turning to the fixed cost of production, we can without loss of generality subtract  $f$  from all payoffs. Hence,  $f$  can be reinterpreted as the *fixed cost of offering an additional product*. This leaves  $\underline{v}$ ,  $\bar{v}$ ,  $\beta$ , and  $f$  as the parameters of interest. Firm 1's payoffs are listed in Table 2, firm 2's payoffs are analogous. Note that we have slightly changed our notation to emphasize that these payoffs also depend on the

intensity of competition  $\beta$ .

1\2	GP	A	B	AB
GP	$\pi_1^*(0, 0; \beta)$	$\pi_1^*(-\bar{v}, -\underline{v}; \beta)$	$\pi_1^*(-\underline{v}, -\bar{v}; \beta)$	$\pi_1^*(-\bar{v}, -\bar{v}; \beta)$
A	$\pi_1^*(\bar{v}, \underline{v}; \beta)$	$\pi_1^*(0, 0; \beta)$	$\pi_1^*(\bar{v} - \underline{v}, \underline{v} - \bar{v}; \beta)$	$\pi_1^*(0, \underline{v} - \bar{v}; \beta)$
B	$\pi_1^*(\underline{v}, \bar{v}; \beta)$	$\pi_1^*(\underline{v} - \bar{v}, \bar{v} - \underline{v}; \beta)$	$\pi_1^*(0, 0; \beta)$	$\pi_1^*(\underline{v} - \bar{v}, 0; \beta)$
AB	$\pi_1^*(\bar{v}, \bar{v}; \beta) - f$	$\pi_1^*(0, \bar{v} - \underline{v}; \beta) - f$	$\pi_1^*(\bar{v} - \underline{v}, 0; \beta) - f$	$\pi_1^*(0, 0; \beta) - f$

Table 2: Payoffs to firm 1 for all possible combinations of product offerings. Firm 1 is row, firm 2 is column.

As an inspection of Table 2 suggests, it is in general impossible to determine the set of SPEs without rather detailed information about the properties of  $\pi_1^*(\Delta_a, \Delta_b; \beta)$ . Nevertheless, we can show that the unique SPE is market segmentation with full-line firms if the degree of fit as well as the degree of misfit are sufficiently low and if the fixed cost of offering an additional product is sufficiently small.

**Proposition 2** *Suppose  $\underline{v} \approx 0$ ,  $\bar{v} \approx 0$ , and  $f \approx 0$ . Then the unique SPE is market segmentation with full-line firms.*

**Proof.** Taken together  $\underline{v} \approx 0$  and  $\bar{v} \approx 0$  ensure that  $\Delta_a \approx \Delta_b$  for all possible combinations of product offerings in Table 2. Hence, Proposition 1 applies and we have  $\frac{\partial \pi_1^*}{\partial \Delta_a} > 0$  and  $\frac{\partial \pi_1^*}{\partial \Delta_b} > 0$ . In light of  $\underline{v} < 0 < \bar{v}$  firm 1 is seen to maximize its gross profit by offering both  $A$  and  $B$  irrespective of firm 2's product offerings. Given that the fixed cost of offering an additional product  $f$  is sufficiently small, offering a full line is thus the dominant strategy, and the claim follows. ■

To better understand what is going on, suppose firm 2 offers both  $A$  and  $B$ . Then there is no point for firm 1 to offer  $GP$  because both segments of consumers can buy their favorite product from firm 2. Moreover, unlike models of spatial competition such as Martinez-Giralt & Neven (1988), consumers have idiosyncratic preferences over brands in our setup. Hence, head-on competition can still generate positive

profits. While these profits are small, so is the fixed cost of offering an additional product, and it is best for firm 1 to also offer both  $A$  and  $B$ .

**Computation.** Since it is impossible to get closed-form solutions for the pricing subgame, there is not much progress to be made using analytical methods. We therefore use numerical methods and make use of the fact that, in two-player games, the set of equilibria with a given support is convex (possibly empty). Hence, it is feasible to enumerate all equilibria using the following algorithm due to Mangasarian (1964): For each possible support, we check for an equilibrium. If there is one, it is either unique or we can find a finite set of extreme points whose convex hull represents the set of equilibria for that support.<sup>2</sup> A formal description of this algorithm can be found in Section 6.3.1 of McKelvey & McLennan (1996). Mangasarian’s (1964) algorithm has been implemented in Gambit. The payoffs themselves are computed in Matlab. We use the `c05nbf` routine of the NAG toolbox, a Newton method, to solve the system of FOCs.

**Parameterization.** Given a point  $(\beta, f)$  we compute the set of equilibria over a grid of  $25^2$  equidistant points  $(\underline{v}, \bar{v})$  in  $[-1, 0) \times (0, 1]$ , implying  $(\Delta_a, \Delta_b) \in [-2, 2]^2$ . To explore how the intensity of competition affects the set of equilibria, we choose a fairly wide range of values with  $\beta \in \{0.5, 1, 2, 4\}$ . The corresponding maximal elasticity of firm 1’s demand with respect to firm 2’s price decreases from 3.16 over 1.89 and 1.39 to 1.18 and the corresponding minimal elasticity increases from 0.32 over 0.53 and 0.72 to 0.85. Next observe that, given  $\beta$ , the set of equilibria remains the same for a sufficiently large fixed cost  $f$ . Because profits from price competition are nonnegative, offering both products eventually becomes a dominated strategy.

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<sup>2</sup>Since almost all finite games have a finite (and odd) number of equilibria (Wilson 1971), we content ourselves with computing the extreme points of the set of the equilibria.

This provides us with an upper bound for  $f$ . To find it, we start with  $f = 0$  and increase  $f$  in increments of 0.1 until the set set of equilibria remains the same.<sup>3</sup>

## 4 Results

Let  $s_n$  denote a (mixed) strategy for firm  $n$ , i.e.,  $s_n$  is a probability distribution over the feasible actions  $GP$ ,  $A$ ,  $B$ , and  $AB$ . In general, there are multiple equilibria for given values of  $\underline{v}$ ,  $\bar{v}$ ,  $\beta$ , and  $f$ . This multiplicity partly reflects the symmetry of the game. In particular, if  $(s_1^*, s_2^*)$  is an equilibrium, then  $(s_2^*, s_1^*)$  is also an equilibrium. Since  $(s_1^*, s_2^*)$  and  $(s_2^*, s_1^*)$  are the same up to a relabelling of firms, we do not distinguish between them in what follows.

There remains a considerable number of equilibria. For example, if  $\underline{v} = -0.96$ ,  $\bar{v} = 0.04$ ,  $\beta = 0.5$ , and  $f = 0.1$ , then there are multiple equilibria in pure strategies: one where one firm offers  $A$  and the other offers  $B$  and one where both firms offer  $GP$ . In addition, there are multiple equilibria in mixed strategies: one where both firms randomize between offering  $A$  and  $B$  with probabilities 0.5 and 0.5, one where one firm randomizes between offering  $GP$  and  $A$  with probabilities 0.99 and 0.01 and the other firm randomizes between offering  $GP$  and  $B$  with probabilities 0.99 and 0.01, and one where both firms randomize between offering  $GP$ ,  $A$ , and  $B$  with probabilities 0.93, 0.04, and 0.04.<sup>4</sup>

**Pareto-undominated equilibria.** We therefore focus on Pareto-undominated equilibria in what follows. This is reasonable because, given a Pareto-dominated equi-

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<sup>3</sup>If  $\beta = 0.5$ , the set of equilibria remains the same for  $f \geq 0.2$ ; if  $\beta = 1$ , the set of equilibria remains the same for  $f \geq 0.3$ ; if  $\beta = 2$ , the set of equilibria remains the same for  $f \geq 0.5$ ; and if  $\beta = 4$ , the set of equilibria remains the same for  $f \geq 0.6$ . The set of Pareto-undominated equilibria remains the same for  $f \geq 0.1$ ,  $f \geq 0.1$ ,  $f \geq 0.2$ , and  $f \geq 0.4$ , respectively.

<sup>4</sup>This multiplicity is reminiscent of Brander & Eaton (1984), where both market segmentation and market interlacing are an equilibrium.

librium, it is in the interest of both firms to coordinate on a different equilibrium. In the above example, the general-purpose equilibrium as well as all mixed-strategy equilibria are Pareto-dominated by the segmentation equilibrium with niche firms. In fact, the gains from coordinating on the Pareto-undominated equilibrium are substantial: each firm's profit increases by between 41% and 138%. This presumably makes market segmentation with niche firms the most likely outcome.

The computations lead to a unique Pareto-undominated equilibrium in pure strategies in all cases. Depending on the values of  $\underline{v}$ ,  $\bar{v}$ ,  $\beta$ , and  $f$ , the Pareto-undominated equilibrium involves either both firms offering the general purpose product,  $(GP, GP)$ , market segmentation with niche firms,  $(A, B)$  (or  $(B, A)$ ), or market segmentation with full-line firms,  $(AB, AB)$ . Figures 2-5 illustrate the results for  $\beta = 0.5$ ,  $f \in \{0, 0.05\}$ , and  $f \geq 0.1$ ;  $\beta = 1$ ,  $f \in \{0, 0.05\}$ , and  $f \geq 0.1$ ;  $\beta = 2$ ,  $f \in \{0, 0.05, 0.1\}$ , and  $f \geq 0.2$ ; as well as  $\beta = 4$ ,  $f \in \{0, 0.05, 0.1, 0.2, 0.3\}$ , and  $f \geq 0.4$ . In each figure we mark the point  $(\underline{v}, \bar{v})$  for which a particular equilibrium occurs as follows:  $(GP, GP)$  is denoted by a blue star,  $(A, B)$  (or  $(B, A)$ ) by a red x-mark, and  $(AB, AB)$  by a green plus.  $\underline{v}$  is denoted as  $vl$ ,  $\bar{v}$  as  $vh$ . Note that Figure 2 is blank for some points  $(\underline{v}, \bar{v})$ . This indicates that there does not exist a NE in all relevant pricing subgames.

**General purpose products.** If the fixed cost of offering an additional product  $f$  is zero, then there is no general-purpose equilibrium. Instead, the market is segmented. If the fixed cost is positive, then  $(GP, GP)$  may be an equilibrium. The tendency towards general purpose products is reinforced by a higher fixed cost.

Both firms forego the possibility of segmenting the market by offering the general purpose product if the degree of fit is not only low (i.e.,  $\bar{v}$  low) but also lower than the degree of misfit (i.e.,  $\bar{v} < |\underline{v}|$ ). In other words, the utility a consumer gains from

buying a product that is targeted at her own segment has to be less than the utility she loses from buying a product that is targeted at the other segment in order for the general-purpose equilibrium to be viable.

Strong brand preferences  $\beta$  facilitate general purpose products. This is in line with our intuition: Since the intensity of competition is low when  $\beta$  is high, avoiding head-on competition through market segmentation becomes less of a concern for firms.

**Market segmentation with niche firms.** A high fixed cost of offering an additional product  $f$  leads to niche firms, whereas strong brand preferences  $\beta$  inhibit niche firms. In fact,  $(A, B)$  (or  $(B, A)$ ) cannot be an equilibrium if the fixed cost is zero and brand preferences are sufficiently high (see  $\beta = 4$  and  $f = 0$  in Figure 5).

$(A, B)$  (or  $(B, A)$ ) may occur in two quite distinct situations. If  $\beta$  is low, it occurs when the degree of fit and the degree of misfit are both *high* (i.e.,  $|\underline{v}|$  and  $\bar{v}$  large; see Figure 2). In this situation, a consumer gains a lot from being able to buy a product that satisfies her needs. By segmenting the market rather than offering a general purpose product, a firm is able to partake in this large gain by charging a higher price. At the same time, however, the consumer loses a lot from not being able to buy a product that satisfies her needs. So why then do firms not offer a full line of products? To see what is going on, consider  $\underline{v} = -0.64$ ,  $\bar{v} = 0.64$ ,  $\beta = 0.5$ , and  $f = 0.05$ . If firm 1 were to deviate by offering  $B$  in addition to  $A$  while firm 2 continues to offer  $B$ , then firm 2 would drop its price from  $p_2^*(-1.28, 1.28; 0.5) = 3.75$  to  $p_2^*(-1.28, 0; 0.5) = 1.26$ . This in turn would force firm 1 to cut its price to  $p_1^*(-1.28, 0; 0.5) = 1.69$ , causing its profit to fall by 50%. More generally, it is precisely because the difference between the utility gain and the utility loss from market segmentation is significant that a firm punishes any infringement on “its” segment by aggressively pushing prices and hence profits down. This keeps a segmentation equilibrium with niche firms in place.

In contrast,  $(A, B)$  (or  $(B, A)$ ) occurs when the degree of fit and the degree of misfit are both *low* (i.e.,  $|\underline{v}|$  and  $\bar{v}$  are low) provided that  $\beta$  is high (see Figure 5). This may seem surprising at first glance. Recall from Table 2 that firm 1's profit is  $\pi_1^*(0, 0; \beta)$  under head-on competition and  $\pi_1^*(\bar{v} - \underline{v}, \underline{v} - \bar{v}; \beta)$  under  $(A, B)$ . Because  $\underline{v} \approx 0$  and  $\bar{v} \approx 0$  imply  $\pi_1^*(\bar{v} - \underline{v}, \underline{v} - \bar{v}; \beta) \approx \pi_1^*(0, 0; \beta)$ , there is little to be gained from segmenting the market in this situation. Yet, as long as the gain is not zero, niche firms arise.

Lastly, combining the above considerations, if  $\beta$  and, in addition,  $f$  is intermediate,  $(A, B)$  (or  $(B, A)$ ) occurs when the degree of fit and the degree of misfit are both high and when they are both low (see  $\beta = 1$  and  $f = 0.05$  in Figure 3 and  $\beta = 2$  and  $f \in \{0.05, 0.1\}$  in Figure 4).

**Market segmentation with full-line firms.** In line with Proposition 2, market segmentation occurs via full-line firms if the degree of fit as well as the degree of misfit are sufficiently low (i.e.,  $\bar{v} \approx 0$  and  $\underline{v} \approx 0$ ) and if the fixed cost of offering an additional product is sufficiently small (i.e.,  $f \approx 0$ , see top left panels of Figures 2-5). A high fixed cost of offering an additional product  $f$ , by contrast, inhibits full-line firms. In fact,  $(AB, AB)$  cannot be an equilibrium if the fixed cost is sufficiently high. Strong brand preferences  $\beta$  make head-on competition more profitable and thus facilitate full-line firms.

Market segmentation with full-line firms takes up the remaining part of  $(\underline{v}, \bar{v})$ -space. In particular, if  $\beta$  and  $f$  are intermediate,  $(AB, AB)$  occurs when either the degree of fit or the degree of misfit is high whereas the other is low (see again Figures 3 and 4).

**Discussion.** Table 3 summarizes the four key determinants of market segmentation. It lists conditions under which in equilibrium both firms forego the possibility of segmenting the market by offering a general purpose product,  $(GP, GP)$ , market segmentation occurs via niche firms,  $(A, B)$  (or  $(B, A)$ ), or market segmentation occurs via full-line firms,  $(AB, AB)$ .

	general purpose products	market segmentation with niche firms	market segmentation with full-line firms
competition	soft	fierce	soft
fixed cost	high	high	low
degree of fit	degree of fit lower	either both high	one high,
degree of misfit	than degree of misfit	or both low	the other low

Table 3: Key determinants of market segmentation.

A higher intensity of competition makes head-on competition less profitable and thus gives firms an incentive to specialize. If the intensity of competition is low, on the other hand, a firm may find it in its best interest to duplicate the product offerings of its rival. If the fixed cost of offering an additional product is low, this leads firms to compete with a full line of products, whereas they compete with general purpose products if the fixed cost is high.

$(GP, GP)$  and  $(AB, AB)$  entails head-on competition and  $(AB, AB)$ , in addition, multiproduct firms. This result stands in marked contrast to models of spatial competition, where price competition gives rise to Hotelling's principle of maximum differentiation and, *de facto*, prevents multiproduct firms. Hence, by not allowing consumers to have preferences over brands the existing models of multiproduct firms (e.g., Martinez-Giralt & Neven 1988) miss an important part of the story.

What role does the tradeoff between fit and misfit that is inherent in market segmentation play? Both firms forego the possibility of segmenting the market by offering the general purpose product if the degree of fit is not only low but also lower

than the degree of misfit. Otherwise, the market is segmented in equilibrium. Niche firms result either when firms have a lot to gain from specializing or when they have too little to gain. The latter situation arises if the degree of fit and the degree of misfit are low and the former if they are high. What prevents a firm from offering a full line of products is that its rival responds to such an infringement by aggressively pushing prices and hence profits down. If either the degree of fit or the degree of misfit is high while the other is low, full-line firms result.

## 5 Conclusions

This paper contributes to the existing literature by considering the tradeoff between the positive and the negative aspects of market segmentation, fit and misfit, that multiproduct firms face when deciding on the number and type of products to offer. We show that firms' market segmentation strategies are influenced by four factors, namely the intensity of competition, the fixed cost of offering an additional product, the degree of fit, and the degree of misfit.

Compared to previous work, which either concludes that there can be no multiproduct firms at all or that firms have a strong incentive to produce general purpose products and that therefore the degree of product differentiation in the marketplace is quite low, our model accounts for a variety of outcomes consistent with the market segmentation strategies that different firms in different industries pursue.

Future work should look into generalizing the model presented here. In its present form, our model implies that gross profits (before fixed costs are subtracted) are the same when both firms offer the general purpose product and when they both offer full lines. This may be relaxed by allowing consumers to opt for a no-purchase alternative. Next, a firm has no incentive to target one segment of consumers and, in addition,

offer the general purpose product because both segments are of the same size and the fixed cost of production depends on the number of products but not on their type. If either of these assumptions is relaxed, then a firm may choose to partially segment the market. More generally, the model should be extended to more than two firms, two segments of consumers, and a correspondingly larger set of possible product offerings.

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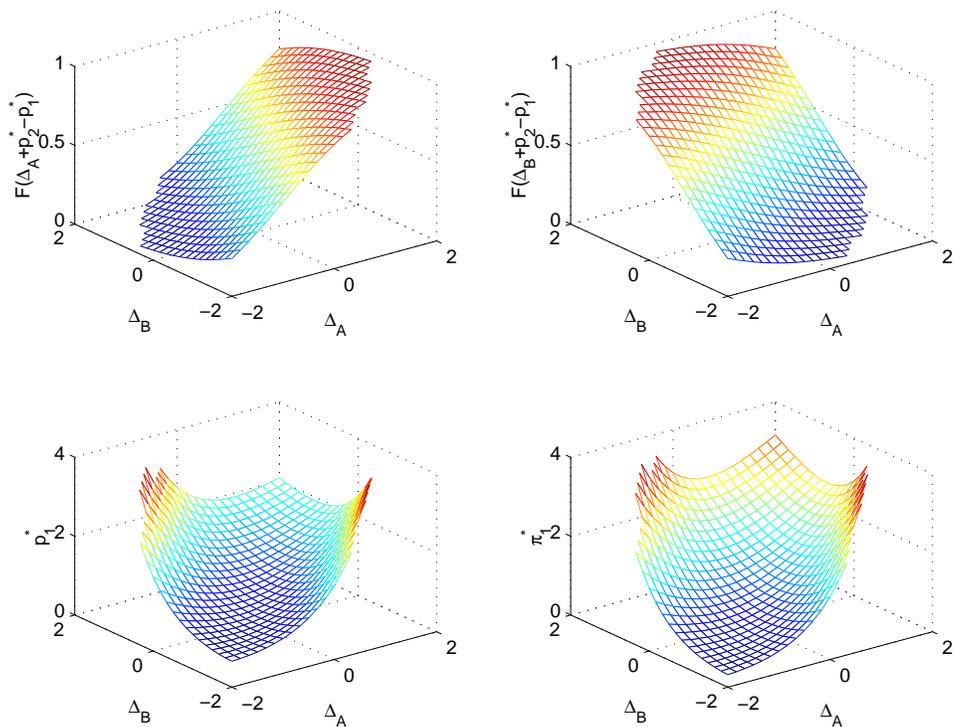


Figure 1: Equilibrium market shares of firm 1 in segments  $a$  and  $b$ ,  $F(\Delta_a + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$  and  $F(\Delta_b + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$ , equilibrium price of firm 1,  $p_1^*(\Delta_a, \Delta_b)$ , and equilibrium profit of firm 1,  $\pi_1^*(\Delta_a, \Delta_b)$ .

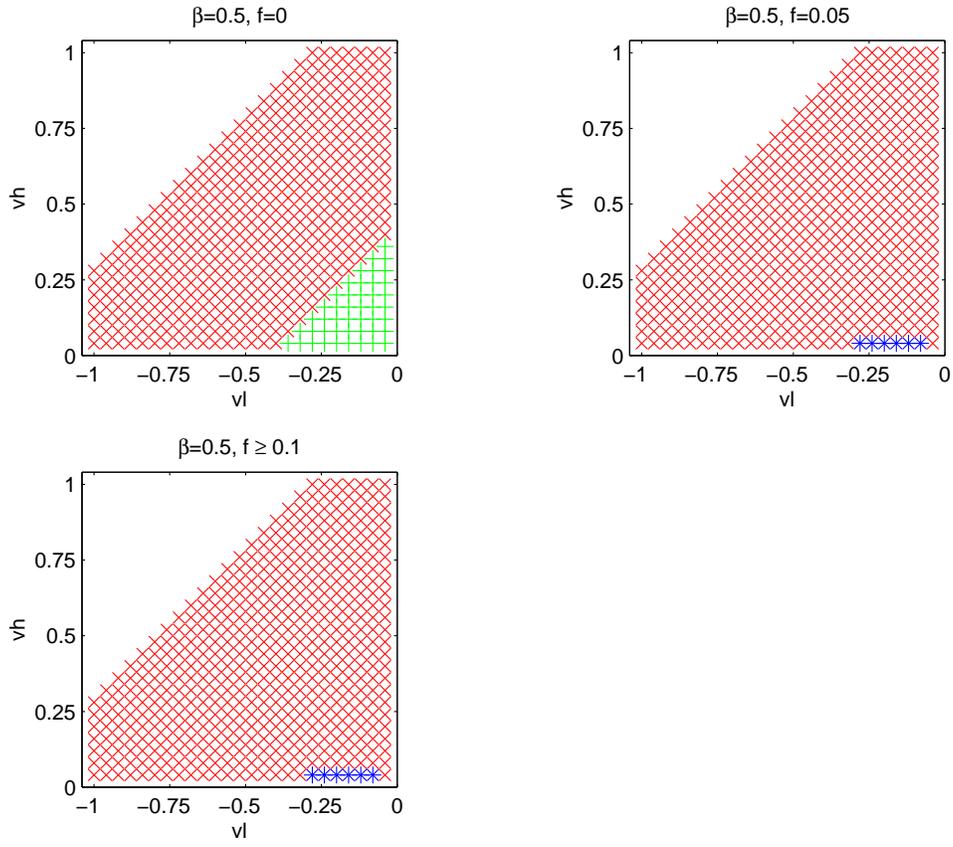


Figure 2: Pareto-undominated equilibria for  $\beta = 0.5$ ,  $f \in \{0, 0.05\}$ , and  $f \geq 0.1$ .  $(GP, GP)$  is denoted by a blue star,  $(A, B)$  (or  $(B, A)$ ) by a red x-mark, and  $(AB, AB)$  by a green plus.

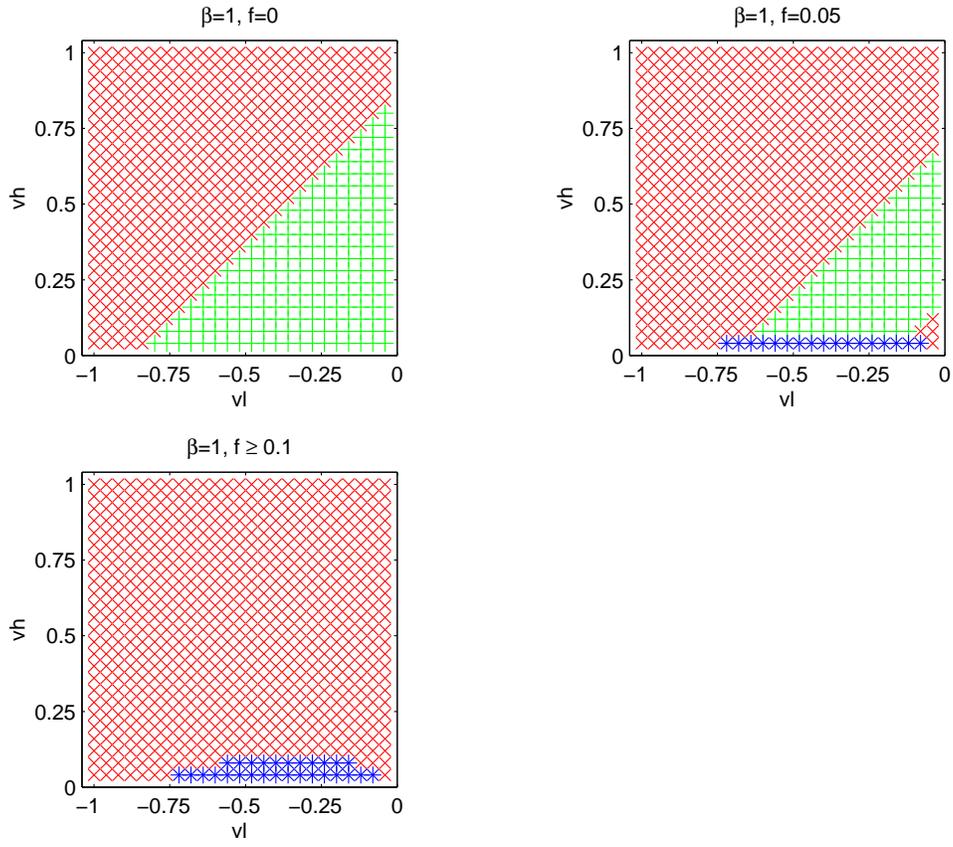


Figure 3: Pareto-undominated equilibria for  $\beta = 1$ ,  $f \in \{0, 0.05\}$ , and  $f \geq 0.1$ .  $(GP, GP)$  is denoted by a blue star,  $(A, B)$  (or  $(B, A)$ ) by a red x-mark, and  $(AB, AB)$  by a green plus.

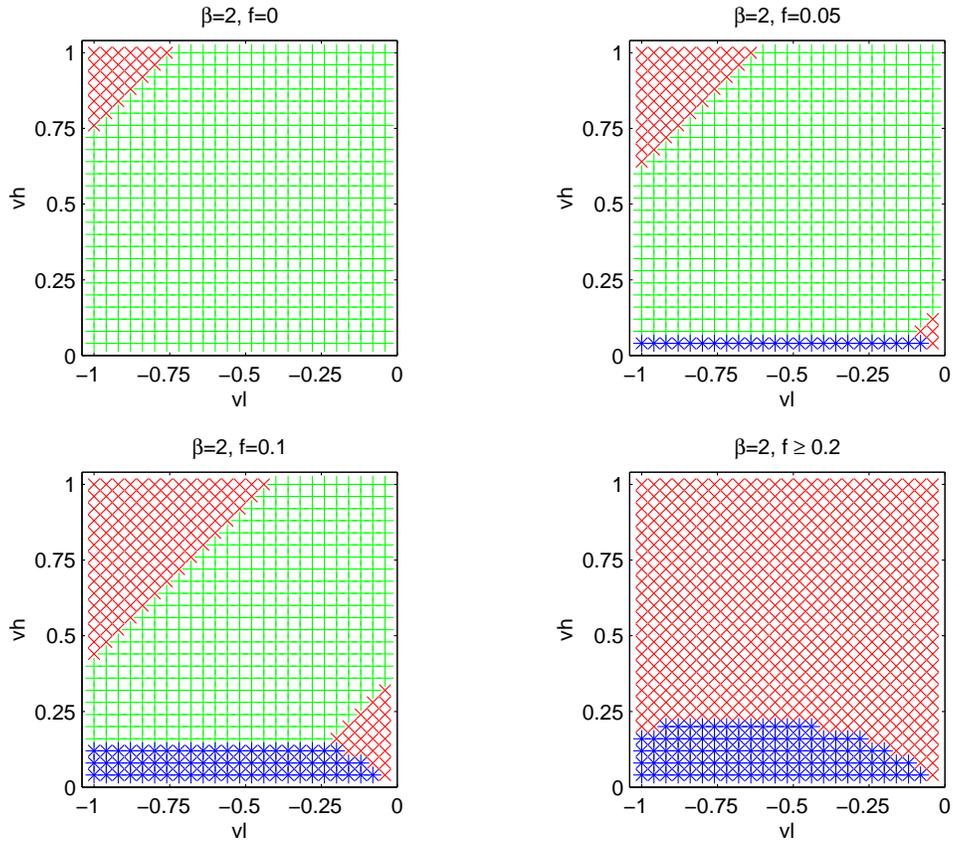


Figure 4: Pareto-undominated equilibria for  $\beta = 2$ ,  $f \in \{0, 0.05, 0.1\}$ , and  $f \geq 0.2$ .  $(GP, GP)$  is denoted by a blue star,  $(A, B)$  (or  $(B, A)$ ) by a red x-mark, and  $(AB, AB)$  by a green plus.

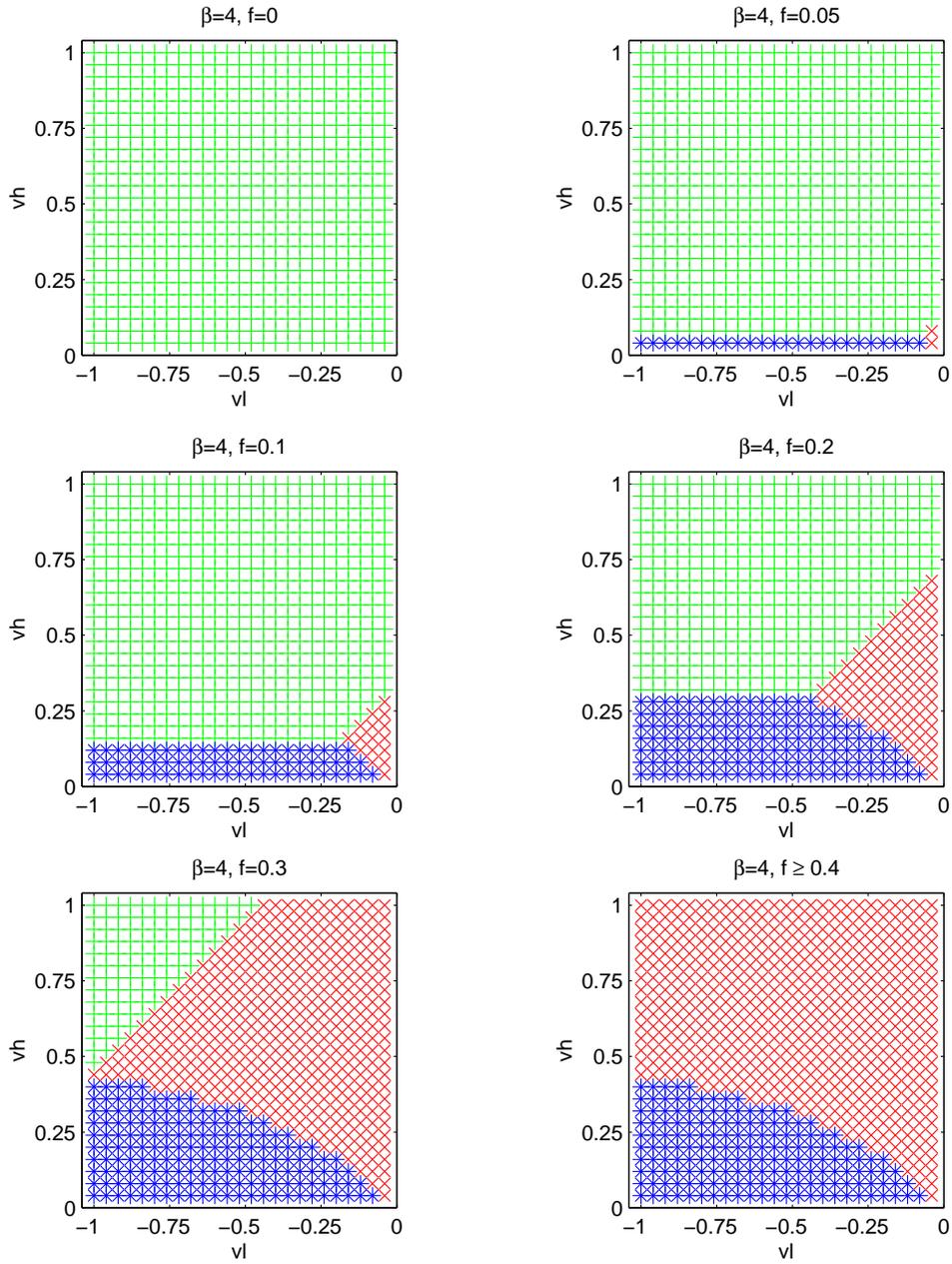


Figure 5: Pareto-undominated equilibria for  $\beta = 0.5$ ,  $f \in \{0, 0.05, 0.1, 0.2, 0.3\}$ , and  $f \geq 0.4$ .  $(GP, GP)$  is denoted by a blue star,  $(A, B)$  (or  $(B, A)$ ) by a red x-mark, and  $(AB, AB)$  by a green plus.